Pressure Drop in Air Piping Systems

Series of Technical White Papers from Ohio Medical Corporation
TERMINOLOGY
ACFM – Actual Cubic Feet per Minute
CFM – Cubic Feet per Minute
GPM – Gallons per Minute

NOMENCLATURE
Constants:
g – Acceleration Due to Gravity (32.2 ft./sec^2)
γ - Specific Weight (lb/ft^3)
ρ - Density of Fluid (slugs/ft^3)
µ - Dynamic Viscosity (lb*s/ft^2)
ε - Equivalent Roughness (.0005 ft. for Galvanized Pipe)
K_L – Loss Coefficient for Fittings (Found in Industrial Literature or College Text)

Variables:
z_n – Height at Position “n” (ft.)
V - Velocity of Fluid (ft/sec.)
D - Diameter of Pipe (in. or ft.)
A – Cross Sectional Area of Pipe (in.^2)
L – Pipe Length (ft.)
h_L – Head Loss (ft.)
p_n – Pressure at Node “n”
f – Friction Factor
INTRODUCTION

Ever since the development of piping systems throughout civilization there has been the need to analyze pipe size for optimal flow. In 1738 Daniel Bernoulli had developed an equation to represent all variables within a piping system, as shown below:

\[
\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L
\]  

From Bernoulli’s equation, we can see how pressure, velocity and position relate to one another, and many derivations can be created from this equation. Depending on the assumptions made and under certain conditions, some of the variables become negligible and can be removed to simplify the equation, and result in a simpler solution. The nature of this paper is to solve for the pressure drop (or commonly referred to the back pressure) which is essentially the pressure difference between two points in a piping system. The pressure drop is critical when sizing pipe. A pump that is integrated within a piping system is designed such that it will withstand certain forces at the inlet and exhaust. If the pump is subjected to forces greater than the ones prescribed, there is a high potential for damage to the internal pump components, and thus the designed flow will be affected. This paper will focus on the flow of air, although it can be shown that other fluids can also be modeled using the same methodology. Like other media that flows within a piping system, air has its own characteristics which benefit and hinder the process.

There are numerous ways in which to solve for the pressure drop of piping system. That is to say, one can incorporate the use of a computer with a plethora of software available. However, the underlying equations used in these programs follow the same fundamental laws of physics found in any collegiate Fluid Mechanics textbook. In addition, sound engineering judgment should be used when sizing pipe. While a cost-effective solution may look good on the bottom line, a safe and reliable system should have precedence in any design.

For our discussion the following assumptions can be made:

Assumptions:
1. Air Flow will be turbulent.
2. The temperature for the ambient air will be 70°F
3. Air flow will be defined using ACFM.

With these assumptions we can model our system.
BACKGROUND:

As we all have either experienced, or heard about, the phenomenon referred to as turbulence. Essentially turbulence is a random positioning of the flow of air. Whereas in a laminar condition, the flow of air is uniform and follows a smooth, organized path. For air piping, we assume that the air flow will be turbulent due to surface randomness in the piping fabrication and/or power fluctuation in the air source equipment. A visual depiction of laminar and turbulent flow is shown below:

![Laminar Flow Diagram](image1)

![Turbulent Flow Diagram](image2)

To see if the flow of air will be turbulent or laminar, we solve for a parameter referred to as the Reynolds number. The Reynolds number is a dimensionless number which is obtained from the following equation:

\[
\text{Re} = \frac{\rho \times V \times D}{\mu}
\]  

(2)

The conditions for if the flow is turbulent or laminar is as follows:

If \( \text{Re} < 2100 \) then the flow is Laminar
If \( \text{Re} > 4000 \) then the flow is Turbulent
If \( 2100 < \text{Re} < 4000 \) then the flow is classified as in Transition.
To determine if the flow rate is laminar or turbulent, the Reynolds number should be calculated. The combination of the Reynolds number, equivalent roughness and pipe diameter we can determine the friction factor from the Moody chart. (Moody charts can be found in industrial literature and in college text books.) The friction factor is used in the equations below.

**OTHER EQUATIONS:**

Some of the other equations used to determine the pressure drop are in this section. We can solve for the velocity of the flow by dividing the ACFM by the cross sectional area:

\[
Velocity \ (ft/s) = \frac{Volumetric\ Flow\ Rate (ft^3/s)}{Area (ft^2)}
\]  

(3)

Where the area is solved by:

\[
A = \pi \cdot r^2
\]

(4)

Once the velocity is found, we can then check to see if the flow is turbulent, or laminar, by utilizing the Reynolds equation (as mentioned previously).

The pressure drop in a piping system can be broken down into two (2) equation forms:

1. Major Pressure Losses (Pipe Losses)
2. Minor Pressure Losses (Losses through fittings, valves, etc.)

The Equation for **Major Losses**:

\[
p_1 - p_2 = f \frac{l}{D} \frac{1}{2} \rho V^2
\]

(5)

The Equation for **Minor Losses**:

\[
p_1 - p_2 = \gamma \sum h_L
\]

(6)

Where \( h_L \) is found by:

\[
h_L = K_L \frac{V^2}{2g}
\]

(7)

And the constant \( K_L \) is found in tables of either college text, or industrial references. Combining Equations (6) and (7) we can solve the pressure differential directly:

\[
p_1 - p_2 = \gamma \sum K_L \frac{V^2}{2g}
\]

(8)
Once the pressure drop has been calculated for the pipe length and all of the fittings/valves, the total pressure drop can be found by the summation of all components.

In Equation form:

\[ P_{\text{Total}} = P_{\text{Fittings}} + P_{\text{Pipe}} + P_{\text{Valves}} \]  \hspace{1cm} (9)

**COMPRESSIBLE VERSUS INCOMPRESSIBLE:**

The reader might be wondering that since air is a gas, shouldn’t the flow be characterized as compressible? The answer to this question is dependent on many conditions. Depending on the length of the pipe and the complexity of the arrangement of fittings and valves, the pressure drop may, or may not, be small relative to the initial pressure. If the pressure drop is small enough, then you can assume the fluid is incompressible. Otherwise, the flow is Compressible, and complicates the analysis. An example to find the pressure ratio is as follows:

We have a pipe length of 7’-0” and the pressure drop should be no greater than 1.0 psi per 7’-0”. The pressure at the beginning is 14.7 psi.

\[ \frac{p_1 - p_2}{p_1} \]  \hspace{1cm} (10)

\[ \frac{\left( \frac{1 \text{ psi}}{7 \text{ ft}} \right) (7 \text{ ft})}{14.7 \text{ psi}} = .068 = 6.8\% \]

This ratio is small enough to assume an incompressible flow. Sound judgment and experience should be used when applying this equation. In different industries, different values are used to make the difference.
EXAMPLE:

Given: A Squire-Cogswell S750TR-T2 system needs to have an exhaust line sized properly. The customer needs to pipe the system to the outside the building. The customer knows that there will be 100 ft. of pipe, three 90° elbows, and two 45° elbows. All Piping will be galvanized ($\varepsilon = 0.0005$ ft.). Air temperature is 60°F and atmospheric pressure is 14.7 psi.

Find: The proper exhaust pipe diameter for this system.

Known: The given flow rate is 163.8 ACFM per pump. The exhaust for the pump is 1-1/2”. (It is always advisable to check with the manufacturer of the pump for a back pressure allowance. Depending on the manufacturer of the pump, the allowable back pressure may vary.) The back pressure should not be greater than 1 psi.

Solution:

First we solve for the major losses-

Find the velocity of the fluid from the flow rate using equation (3):

$$V = \frac{163.8\text{ ft}^3}{\text{min}} \times \frac{1\text{min}}{60\text{sec}} = 2.73\text{ ft}^3\text{ sec}$$

$$V = 2.73\text{ ft}^3\text{ sec} \times \frac{1}{\text{Cross Sectional Area of Pipe}}$$

Let’s assume a 1-1/2” (0.0625 ft.) Cross Sectional Diameter:

Therefore:

$$V = 2.73\text{ ft}^3\text{ sec} \times \frac{1}{(0.0625\text{ ft})^2 \times 3.14} = 222.6\text{ ft}^3\text{ sec}$$

The next variable we look for is the Reynolds Number using equation (2):

$$\text{Re} = \frac{\rho V D}{\mu}$$

$$\text{Re} = \left[ \frac{0.00238\text{ slugs}}{\text{ft}^3} \right] \left[ \frac{222.6\text{ ft}}{s} \right] \left( \frac{0.125\text{ ft.}}{s} \right) = 176.471$$
Once the Reynolds number is found then the flow can be determined as either turbulent or laminar. In this example, the flow is turbulent. The frictional factor can be found from knowing the Reynolds number, relative roughness and diameter of the pipe.

From the Moody Chart \( f = 0.029 \)

Using equation (5) to solve for the major losses:

\[
p_1 - p_2 = 0.029 \left( \frac{100 \text{ ft.}}{0.125 \text{ ft.}} \right) \frac{1}{2} \left( 0.00238 \frac{\text{slugs}}{\text{ft}^3} \right) \left( 222.6 \frac{\text{ft}}{\text{sec}} \right)^2
\]

\[
p_1 - p_2 = 1368 \frac{\text{lb}}{\text{ft}^2}
\]

\[
p_1 - p_2 = 1368 \frac{\text{lb}}{\text{ft}^2} \times \frac{1\text{ ft}^2}{144\text{ in}^2}
\]

\[
\Delta p = 9.49 \text{ psi}
\]

Second we solve for the minor losses (i.e. through fittings and valves) using equation (8):

\[
p_1 - p_2 = \gamma \sum K_L \frac{V^2}{2g}
\]

\[
p_1 - p_2 = 0.0765 \frac{\text{lb}}{\text{ft}^3} \left[ 3 \left( \frac{222.6 \frac{\text{ft}}{\text{s}}}{2 \left( 32.2 \frac{\text{ft}}{\text{s}^2} \right)} \right)^2 + 2 \left( \frac{222.6 \frac{\text{ft}}{\text{s}}}{2 \left( 32.2 \frac{\text{ft}}{\text{s}^2} \right)} \right)^2 \right]
\]

\[
\Delta p = 359 \frac{\text{lb}}{\text{ft}^2}
\]

\[
\Delta p = 359 \frac{\text{lb}}{\text{ft}^2} \times \frac{1\text{ ft}^2}{144\text{ in}^2}
\]

\[
\Delta p = 2.50 \text{ psi}
\]

\[ p_{Total} = p_{Fittings} + p_{Pipe} \]
\[ p_{\text{Total}} = 9.49 \, \text{psi} + 2.50 \, \text{psi} \]

\[ \Delta p_{\text{Total}} = 11.99 \, \text{psi} \]

As we can see from this solution, the pressure drop is much higher than what should be observed in the pumps. Therefore, other iterations of the example are shown in Table 1:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Pipe Size (in.)</th>
<th>Major Losses Pressure Drop in Pipe (psi)</th>
<th>Minor Losses Pressure Drop in Fittings (psi)</th>
<th>Total Pressure Drop (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1/2</td>
<td>9.49</td>
<td>2.20</td>
<td>11.99</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.24</td>
<td>0.14</td>
<td>0.38</td>
</tr>
<tr>
<td>3</td>
<td>2 1/2</td>
<td>0.64</td>
<td>0.28</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Table 1

EXPERIMENT:

The above theory was tested in our lab to see if the data correlated with the solutions. The experiment was set up as seen in the following pictures:
Pressure Measuring Device:
Merical DP2000I (Accuracy 0.05%%)

Pump:
45 lpm Volumetric Free Flow, 12 VDC, 4.0 Amp

Pipe Size:
1/8” (.269ID)

Fittings:
1/8” Tee’s (Qty. 2)
1/8” Pipe Nipples (as appropriate)
1/8” Pipe Couplings (as appropriate)

Conditions for the test:

Ambient conditions at Sea Level:

Temperature - 70°F
Pressure – 14.7 PSIA

Flow Rate:

45 lpm (Approximately 1.6 ACFM)

The pressure drop was measured at different pipe lengths. Refer to the data below.

<table>
<thead>
<tr>
<th>Length</th>
<th>Major Losses Calculated (psi)</th>
<th>Minor Losses Calculated (psi)</th>
<th>ΔP Calculated (psi)</th>
<th>ΔP Measured (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6”</td>
<td>0.04</td>
<td>Tee Branch Flow: 0.05(2) = 0.10 Elbow: 0.04(1) = 0.04</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>12”</td>
<td>0.09</td>
<td>Tee Branch Flow: 0.05(2) = 0.10 Elbow: 0.04(1) = 0.04 1 Coupling: 0.002(1) = 0.002</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>24”</td>
<td>0.17</td>
<td>Tee Branch Flow: 0.05(2) = 0.10 Elbow: 0.04(1) = 0.04 3 Couplings: 0.002(3) = 0.006</td>
<td>0.32</td>
<td>0.30</td>
</tr>
<tr>
<td>36”</td>
<td>0.26</td>
<td>Tee Branch Flow: 0.05(2) = 0.10 Elbow: 0.04(1) = 0.04 5 Couplings: 0.002(5) = 0.010</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>48”</td>
<td>0.35</td>
<td>Tee Branch Flow: 0.05(2) = 0.10 Elbow: 0.04(1) = 0.04 7 Couplings: 0.002(7) = 0.014</td>
<td>0.49</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 2
As we can see from the data above, the calculated and measured data points correlate with each other. The average difference is 0.03 psi between the calculated and measured data points. The non-linear behavior of the measured values is attributed to the random behavior of air. From Figure 3, one can see that the pressure data separates from the 12” point. The separation of data points can be attributed to leaks or irregularities in the pipe (i.e. burrs, surface irregularities from galvanizing etc.) The leaks in the piping system will decrease the flow rate, this in turn decreases the pressure. It is also interesting to note that as the pipe length increases, so does the pressure. This is both demonstrated in the measured and calculated values.
CONCLUSION:

Optimization of piping is essential in today’s new construction of building systems. Due to ever increasing costs of steel and copper, there is no other alternative but to take a closer look at the piping system. In the past, a “rule of thumb” gave a large margin of safety. However, the Engineer, Contractor and Architect must understand that the margin of safety can be held while decreasing the excess size of the pipe. As long as the pressure drop is within the pump manufacturer’s specifications, the performance will not be affected.

The pressure drop in the piping system directly affects the performance of the pump. Essentially the pressure drop produces additional forces that directly and indirectly affect the internal parts. It should be noted that having an excess size diameter of pipe can be detrimental as well (i.e. oversized). If the area is increased, the excess (stagnant) air will act as an obstruction to the flow and therefore create a greater turbulence. Another detrimental effect, besides damage to the internal components of the pump, is the reduction of air capacity the pump removes. The back pressure acts as an obstruction to the flow, therefore hindering capacity.

The example could be solved in different ways. In fact we could have different pipe sizes in this line. That is to say, we could have calculated for a run 50’-0” of 1 ½” Dia. and 50’-0” of 3” Dia. The calculation above is good for approximating the pipe size for the entire length of the pipe, and it is also good for solving for smaller sections of pipe. In addition, this paper makes lot of assumptions regarding the condition of the air flow and the ambient conditions. It can be argued that a more detailed analysis would be appropriate, and in other cases a more general solving approach would be appropriate.

As shown in the example, there are many steps that need to be followed to give us an optimized pipe size. There are many opportunities to overlook error by using these equations. Therefore, it is recommended to use sound industry judgment when solving for the pressure drop and interpreting the results.